Crossed modules over inverse semigroups, crossed module extensions and their cohomological interpretation

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We introduce the notion of a crossed module over an inverse semigroup which generalizes the notion of a module over an inverse semigroup in the sense of Lausch [1], as well as the notion of a crossed module over a group in the sense of Whitehead [2] and Maclane [3]. With any crossed S-module A we associate a 4-term exact sequence of inverse semigroups $A \xrightarrow{i} N \xrightarrow{\beta} S \xrightarrow{\pi} T$, which we call a crossed module extension of A by T. We then introduce the so-called admissible crossed module extensions and show that equivalence classes of admissible crossed module extensions of A by T are in a one-to-one correspondence with the elements of the cohomology group $H^3_{\leqslant}(T^1, A^1)$, whenever T is an F-inverse monoid.

This is a joint work [4] with Mikhailo Dokuchaev (Universidade de São Paulo) and Mayumi Makuta (Universidade de São Paulo).

References

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