Partial actions and Galois theory of commutative rings

Héctor Pinedo Tapia
School of Mathematics, Industrial university of Santander – Colombia

The concept of a Galois extension of commutative rings was introduced by Auslander and Goldman [1], in which they laid the foundations for separable extensions and defined the Brauer group of a commutative ring. Later, in [2], Chase, Harrison and Rosenberg developed a Galois theory of commutative rings by giving several equivalent definitions of a Galois extension and specifying, to the case of a Galois extension, the Amitsur cohomology seven terms exact sequence, given by Chase and Rosenberg in [2]. The Chase-Harrison-Rosenberg sequence can be viewed as a common generalization of the two most fundamental facts from Galois cohomology of fields: Hilbert's Theorem 90 and the isomorphism of the relative Brauer group with the second cohomology group of the Galois group. When working with abelian groups and having the purpose of presenting a Kummer's theory for commutative rings, Harrison constructed in [3] the group of the isomorphism classes of abelian G-extensions of a commutative ring. Since then much attention have been paid to the sequence and its parts subject to more constructive proofs, generalizations of Harrison's group and analogs in various contexts.

Another point of view is to replace global actions by partial ones. The latter are becoming an object of intensive research and have their origins in the theory of operator algebras, and were initiated by Exel in [4]. In the algebraic context, a partial action of a group G on a ring R consists of a family of ring isomorphisms $\alpha : \{\alpha_g : D_{g^{-1}} \to D_g\}_{g\in G}$ such that any D_g is an ideal of R, α_e is the identity map of R and α_{gh} extends $\alpha_g \circ \alpha_h, g, h \in G,$. The development of a Galois theory of partial actions was initiated in [5] stimulating a growing algebraic activity around partial actions, while the partial cohomology of groups was introduced and studied in [6].

Having at hand partial Galois theory and partial group cohomology, we may ask now what would be the analog of the Chase-Harrison-Rosenberg exact sequence in the context of a partial Galois extension of commutative rings and to explore Harrison's construction to the context of partial Galois extensions. This talk is based on the papers [7], [8] and [9] where these questions were answered. The interested audience may find some other extension of Chase-Harrison-Rosenberg sequence in [10] and [11].

References

[1] M. Auslander and O. Goldman, "The Brauer group of a commutative ring," *Trans. Amer. Math. Soc.*, vol. 97, pp. 367–409, 1960, ISSN: 0002-9947. DOI: 10.2307/1993378.

[2] S. U. Chase, D. K. Harrison, and A. Rosenberg, "Galois theory and Galois cohomology of commutative rings," *Mem. Amer. Math. Soc.*, vol. 52, pp. 15–33, 1965, ISSN: 0065-9266.

[3] D. K. Harrison, "Abelian extensions of commutative rings," *Mem. Amer. Math. Soc.*, vol. 52, pp. 1–14, 1965, ISSN: 0065-9266.

[4] R. Exel, "Circle actions on C^*-algebras, partial automorphisms, and a generalized Pimsner-Voiculescu exact sequence," *J. Funct. Anal.*, vol. 122, no. 2, pp. 361–401, 1994, ISSN: 0022-1236. DOI: 10.1006/jfan.1994.1073.

[5] M. Dokuchaev, M. Ferrero, and A. Paques, "Partial actions and Galois theory," *J. Pure Appl. Algebra*, vol. 208, no. 1, pp. 77–87, 2007, ISSN: 0022-4049. DOI: 10.1016/j.jpaa.2005.11.009.

[6] M. Dokuchaev and M. Khrypchenko, "Partial cohomology of groups," *J. Algebra*, vol. 427, pp. 142–182, 2015, ISSN: 0021-8693. DOI: 10.1016/j.jalgebra.2014.11.030.

[7] D. Bagio, A. Cañas, V. Marín, A. Paques, and H. Pinedo, "The commutative inverse semigroup of partial abelian extensions," *Comm. Algebra*, To appear.

[8] M. Dokuchaev, A. Paques, and H. Pinedo, "Partial Galois cohomology and related homomorphisms," *Q. J. Math.*, vol. 70, no. 2, pp. 737–766, 2019, ISSN: 0033-5606. DOI: 10.1093/qmath/hay062.

[9] M. Dokuchaev, A. Paques, H. Pinedo, and I. Rocha, "Partial generalized crossed products and a seven-term exact sequence," *J. Algebra*, vol. 572, pp. 195–230, 2021, ISSN: 0021-8693. DOI: 10.1016/j.jalgebra.2020.12.014.

[10] D. Crocker, A. Kumjian, I. Raeburn, and D. P. Williams, "An equivariant Brauer group and actions of groups on C^*-algebras," *J. Funct. Anal.*, vol. 146, no. 1, pp. 151–184, 1997, ISSN: 0022-1236. DOI: 10.1006/jfan.1996.3010.

[11] L. El Kaoutit and J. Gómez-Torrecillas, "Invertible unital bimodules over rings with local units, and related exact sequences of groups, II," *J. Algebra*, vol. 370, pp. 266–296, 2012, ISSN: 0021-8693. DOI: 10.1016/j.jalgebra.2012.07.035.