Representations of the restricted enveloping algebra $\mathfrak{u}(\mathfrak{m})$ in characteristic 2.

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Let $\Bbbk$ be an algebraically closed field of characteristic 2 and $\mathfrak{s}$ the unique, up to isomorphism, not restricted simple Lie algebra of dimension 3 over $\Bbbk$ which has basis $\{e, h, f\}$ and bracket $[e, f] = h$, $[e, h] = e$ and $[f, h] = f$. The 2-closure $\mathfrak{m}$ of $\mathfrak{s}$ (that is, $\mathfrak{m}$ is the smallest restricted Lie algebra that contains $\mathfrak{s}$) is a 5-dimensional Lie algebra and its restricted enveloping algebra $\mathfrak{u}(\mathfrak{m})$ is generated by a, b, c with defining relations

$$ef + fe = h, \qquad eh + he = e, \qquad fh + hf = f, \qquad e^4 = f^4 = 0, \qquad h^2 + h = 0.$$

We prove that $\mathfrak{u}(\mathfrak{m})$ is a special biserial algebra and hence it is of tame representation type [1]. The description of all indecomposable modules of a special biserial algebra was given in Proposition 2.3 of [2]. They are either string modules or band modules. Using this description, we present explicitly all families of finite-dimensional indecomposable $\mathfrak{u}(\mathfrak{m})$-modules.

We are interested in the representation theory of $\mathfrak{u}(\mathfrak{m})$ by the following reason. Let $\mathscr{B}(V)$ be the restricted Jordan plane in characteristic 2. Consider the Hopf algebra $H = \mathscr{B}(V)\#\Bbbk\mathbb{Z}_2$ and $D(H)$ the Drinfeld double of H. Then there are a central Hopf subalgebra $\mathbf{R}$ of $D(H)$ and an exact sequence $\mathbf{R} \hookrightarrow D(H) \twoheadrightarrow \mathfrak{u}(\mathfrak{m})$ of Hopf algebras. Therefore, we obtain the forgetful functor ${}_{\mathfrak{u}(\mathfrak{m})}\mathcal{M} \to {}_{D(H)}\mathcal{M}$. This is a joint work with N. Andruskiewitsch, S. D. Flora and D. Flôres.

References

[1] W. Crawley-Boevey, "Tameness of biserial algebras," *Arch. Math. (Basel)*, vol. 65, no. 5, pp. 399–407, 1995, ISSN: 0003-889X. DOI: 10.1007/BF01198070.

[2] B. Wald and J. Waschbüsch, "Tame biserial algebras," *J. Algebra*, vol. 95, no. 2, pp. 480–500, 1985, ISSN: 0021-8693. DOI: 10.1016/0021-8693(85)90119-X.