Bicategories, 2-monads, enriched and internal categories

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For this course a preknowledge on monoidal categories is very helpful. The preliminary plan for the three lectures is as follows.

After recalling the definition of a monoidal category we will introduce bicategories and study some examples of them. We will define pseudofunctors between bicategories and we will introduce 2-monads as lax functors from the trivial bicategory. We will also introduce T-algebras in bicategories (over 2-monads T) and compare them to module categories over tensor categories. We will show how a 2-monad in the bicategory $\mathrm{Span}(C)$ of spans over a category C with pullbacks is an internal category in C (actually, Benaboú defines them this way in his famous paper from 1967).

We will define double categories, originally introduced by Charles Ehresmann in 1963, as categories internal in Cat_1, the category of categories, and also pseudodouble categories, as categories weakly internal in Cat_1, or internal in the 2-category Cat_2 of categories. The latter is a special case of pseudocategories, introduced by Martins, as categories internal to 2-categories. We will state the Strictification Theorem for double categories and study the relation between bicategories and double categories. We will introduce the bicategory $\mathrm{Mat}(C)$ of matrices over a category C will products and illustrate its biequivalence with a sub-bicategory of $\mathrm{Span}(C)$, under certain assumptions.

We will show that 2-monads in $\mathrm{Mat}(C)$ and in $\mathrm{Span}(C)$ are categories enriched, respectively internal in V. We will illustrate the embedding of the category $C-\mathrm{Cat}$ (categories enriched over C) into $\mathrm{Cat}(C)$ (categories internal in C). We will comment on the analogous result for when C is a certain type of tricategory, and illustrate it with the example of tensor categories and module categories over them. We will also give examples of the latter result in lower dimensions.

References

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