Simplicity of Nekrashevych algebras of contracting self-similar groups

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A self-similar group is a group G acting on the Cayley graph of a finitely generated free monoid X^* (i.e., regular rooted tree) by automorphisms in such a way that the self-similarity of the tree is reflected in the group. The most common examples are generated by the states of a finite automaton. Many famous groups, like Grigorchuk's 2-group of intermediate growth are of this form. Nekrashevych associated C^*-algebras and algebras with coefficients in a field to self-similar groups. In the case G is trivial, the algebra is the classical Leavitt algebra, a famous finitely presented simple algebra. Nekrashevych showed that the algebra associated to the Grigorchuk group is not simple in characteristic 2, but Clark, Exel, Pardo, Sims and Starling showed its Nekrashevych algebra is simple over all other fields. Nekrashevych then showed that the algebra associated to the Grigorchuk-Erschler group is not simple over any field (the first such example). The Grigorchuk and Grigorchuk-Erschler groups are contracting self-similar groups. This important class of self-similar groups includes Gupta-Sidki p-groups and many iterated monodromy groups like the Basilica group. Nekrashevych proved algebras associated to contacting groups are finitely presented.

In this talk we discuss a recent result of the speaker and N. Szakacs (Manchester) characterizing simplicity of Nekrashevych algebras of contracting groups. In particular, we give an algorithm for deciding simplicity given an automaton generating the group. We apply our results to several families of contracting groups like Gupta-Sidki groups, GGS groups and Sunic's generalizations of Grigorchuk's group associated to polynomials over finite fields.